3.4 Scalene Triangle

(A triangle with no two sides equal)

Sides of a triangle: a, b, c

Semiperimeter: $p = \frac{a+b+c}{2}$

Angles of a triangle: α, β, γ

Altitudes to the sides a, b, c: h_a, h_b, h_c

Medians to the sides a, b, c: m_a, m_b, m_c

Bisectors of the angles α, β, γ: t_a, t_b, t_c

Radius of circumscribed circle: R

Radius of inscribed circle: r

Area: S

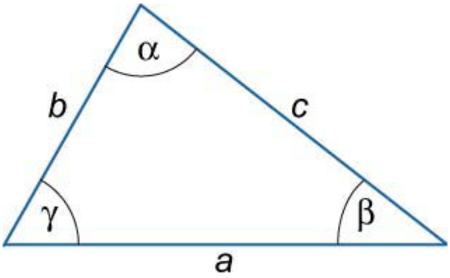

Figure 13.

181. $\alpha + \beta + \gamma = 180°$

182. $a + b > c$,
$b + c > a$,
$a + c > b$.

183. $|a - b| < c$,
$|b - c| < a$,
$|a - c| < b$.

184. Midline

$$q = \frac{a}{2},\ q \parallel a.$$

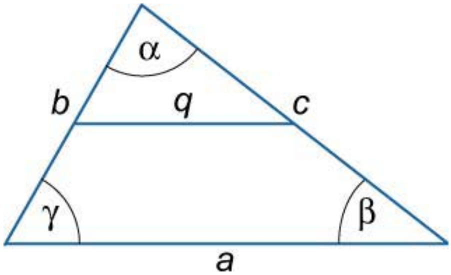

Figure 14.

185. Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha,$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta,$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma.$$

186. Law of Sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R,$$

where R is the radius of the circumscribed circle.

187. $$R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma} = \frac{bc}{2h_a} = \frac{ac}{2h_b} = \frac{ab}{2h_c} = \frac{abc}{4S}$$

188. $$r^2 = \frac{(p-a)(p-b)(p-c)}{p},$$

$$\frac{1}{r} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}.$$

189. $\sin\frac{\alpha}{2}=\sqrt{\frac{(p-b)(p-c)}{bc}}$,

$\cos\frac{\alpha}{2}=\sqrt{\frac{p(p-a)}{bc}}$,

$\tan\frac{\alpha}{2}=\sqrt{\frac{(p-b)(p-c)}{p(p-a)}}$.

190. $h_a=\frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)}$,

$h_b=\frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}$,

$h_c=\frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}$.

191. $h_a=b\sin\gamma=c\sin\beta$,

$h_b=a\sin\gamma=c\sin\alpha$,

$h_c=a\sin\beta=b\sin\alpha$.

192. $m_a^2=\frac{b^2+c^2}{2}-\frac{a^2}{4}$,

$m_b^2=\frac{a^2+c^2}{2}-\frac{b^2}{4}$,

$m_c^2=\frac{a^2+b^2}{2}-\frac{c^2}{4}$.

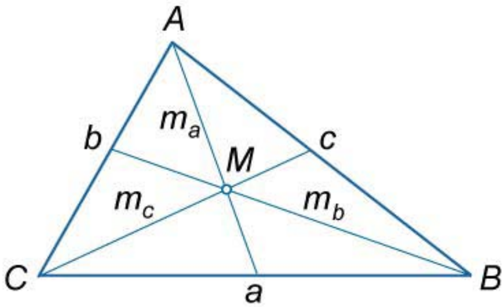

Figure 15.

193. $AM=\frac{2}{3}m_a$, $BM=\frac{2}{3}m_b$, $CM=\frac{2}{3}m_c$ (Fig.15).

194. $t_a^2=\frac{4bcp(p-a)}{(b+c)^2}$,

$t_b^2=\frac{4acp(p-b)}{(a+c)^2}$,

$t_c^2=\frac{4abp(p-c)}{(a+b)^2}$.

195. $S=\frac{ah_a}{2}=\frac{bh_b}{2}=\frac{ch_c}{2}$,

$S=\frac{ab\sin\gamma}{2}=\frac{ac\sin\beta}{2}=\frac{bc\sin\alpha}{2}$,

$S=\sqrt{p(p-a)(p-b)(p-c)}$ (Heron's Formula),

$S=pr$,

$S=\frac{abc}{4R}$,

$S=2R^2\sin\alpha\sin\beta\ \sin\gamma$,

$S=p^2\ \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$.